SOLUTION GUIDE TO MATH GRE FORM 9768

IAN COLEY

The questions for this solution guide can be found here.

Solution 1. (C) Fundamental theorem of calculus:

$$\frac{d}{dx}\int_e^x \log t\,dt = \log x.$$

Solution 2. (D) There is an easy way to get a closed formula for this series.

$$F(2) = 2 + 1/2, \quad f(3) = 2 + 1/2 + 1/2, \ldots \quad F(n) = 2 + (n-1)/2.$$

Hence $F(101) = 52$.

Solution 3. (C) The inverse of a 2×2 matrix should be well known: flip the diagonal and negate the antidiagonal, then divide by the determinant. That gives us (C).

Solution 4. (B) The shaded region given above is

$$I = \int_1^b x^2 - x\,dx.$$

Then

$$0 = \int_0^b x^2 - x\,dx = \int_0^1 x^2 - x\,dx + \int_1^b x^2 - x\,dx \implies I = \int_0^1 x - x^2\,dx.$$

That is now an easy integral to compute. $I = (x^2/2 - x^3/3)\Big|_0^1 = 1/6$.

Solution 5. (E) The graph of $f'(x)$ tells us that we have minima at $x = -3.5$ and $x = 10$ and a maximum at $x = 5$. The only graph that fits that description is (E).

Solution 6. (C) Throughout the function, we have $i = 2^p$. The algorithm terminates when $i \geq k$, and then we print p. We know that $2^{10} = 1024 > 999$, so the p that gets printed is 10.

Solution 7. (B) The general graph $(\sin t, \cos t)$ is a circle, but that is when we take $t \in [0, 2\pi]$. For us, however, we start at $-\pi/2$, at the point $(-1, 0)$, and rotate one-quarter circle to $(0, 1)$.

Solution 8. (E) Luckily we can use a u-substitution here. Let $u = 1+x^2$ so that $du = 2x\,dx$. That makes our integral

$$\int_0^1 \frac{x}{1+x^2}\,dx = \int_1^2 \frac{1}{2u}\,du = (\log u)/2\Big|_1^2 = \frac{\log 2}{2}.$$

This isn't an option by itself, so we move the coefficient inside the log: $\log\sqrt{2}$.

Solution 9. (A) Let $S = \{1, \ldots, k\}$. Then $f(1)$ has k choices, $f(2)$ has $k-1$ choices, etc. That makes $k!$ choices in total if we want the function to be bijective.

Last Updated: July 27, 2021.

Solution 10. (B) A usual sort of problem. Let $\varepsilon > 0$. Then by the continuity of e^x, we know that there exists a $\delta > 0$ such that $|f(x) - 1| < \varepsilon$ whenever $|x| < \delta$. That makes $f(x)$ continuous at $x = 0$ at the very least. However, let $x \neq 0$. Then we know that $|f(x)-1| > 0$, so there exists some $\varepsilon > 0$ such that $|f(x) - 1| > \varepsilon$. Since within any δ-neighbourhood of x there exists both a rational and an irrational number, we will never have continuity.

Solution 11. (A) Suppose that $x > y$. Then $|x - y| = x - y$, so $x + y + |x - y| = 2x$, so the expression is equal to x. If $y > x$, then $|x - y| = y - x$, so the expression is equal to y. That means the expression gives us the maximum.

Solution 12. (C) If B is bounded, then $\sup B$ is a limit point of B. B is certainly not closed, as it does not contain its limit point, and B may open. There must be a sequence in B converging to its supremum, and (E) is the opposite of a limit point.

Solution 13. (A) There are three options: two blue, two red, or two yellow. The odds of getting two blue is $2/8 \cdot 1/7$, and the same for yellow. The odds of getting two red is $4/8 \cdot 3/7$. We obtain

$$\frac{2 + 12 + 2}{56} = \frac{16}{56} = \frac{2}{7}.$$

Solution 14. (C) We can attempt to write this in logical notation

$$\forall s \in \mathbb{R} \exists r \in \mathbb{R}(f(r) > 0 \implies g(s) > 0).$$

Something like that is proper logical notation. Negating everything gives

$$\exists s \in \mathbb{R} \forall r \in \mathbb{R}(g(s) \leq 0 \text{ and } f(r) > 0).$$

That makes the answer (C).

Solution 15. (B) The function is certainly bounded, as $a < f(x) < x < b$ for all $x \in (a, b)$. There again is no reason the function should be nonnegative: let $b < 0$, for example. The function needn't be strictly increasing, though it must start out by increasing, and again there is no reason it should be polynomial. But $f(x)$ must be nonconstant: suppose that $f(x) = c$. We must have $a < c$ because $a < f(x)$. Then we would have that $f((c+a)/2) = c$, but $f((c + a)/2) < (c + a)/2$, a contradiction.

Solution 16. (D) Call those three vectors v_1, v_2, v_3 for simplicity. Suppose that $(1, 2, m, 5) = av_1 + bv_2 + cv_3$. This gives us the system of equations:

$$(1, 2, m, 5) = (c, a + c, a + 2c, a + b).$$

This tells us that $c = 1$, so $a = 1$ as well. That further means that $b = 4$. This proves that $m = 3$.

Solution 17. (E) This is straightforward but pretty annoying. From the value of Δ^2 we have that $\Delta f(3) = 4$. We then have $f(4) = f(1) + \Delta f(1) + \Delta f(2) + \Delta f(3)$, so $f(4) = 5$.

Solution 18. (E) If r is the inner radius, then $1 - r$ is the radius of circle O. We have that $a(r) = \pi(1 - r)^2$ and $A(r) = \pi(1 - r^2)$. The limit therefore becomes

$$\lim_{r \to 1^-} \frac{\pi(1 - r^2)}{\pi(1 - r)^2} = \lim_{r \to 1^-} \frac{1 - r^2}{(1 - r)^2} = \lim_{r \to 1^-} \frac{1 + r}{1 - r} = \infty$$

Solution 19. (B) We can see that a is always the identity element in these groups. Further, all these groups are commutative. In order to confirm these are multiplication tables, we must have a, b, c, d appear in each row and each column exactly once, as left or right multiplication by a fixed group element is a set bijection. Only I satisfies this.

Solution 20. (D) Given that $f(0) = 0$, we can write

$$f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \frac{f(x)}{x}.$$

That means that $f'(0) = L$, and f is differentiable at 0. In particular, f must be continuous there as well, so $\lim_{x\to 0} f(x) = f(0) = 0$.

Solution 21. (D) Let us attempt to figure out what this triangle is. We have that $y' = x/4 + 1/2$, so $y'(0) = 1/2$. That makes the equation of the line $y = x/2 + 1$. That makes our region a triangle of base 2 and height 1, so its area is 1.

Solution 22. (B) The positive integers are not a subgroup of the integers under addition. There are no inverses.

Solution 23. (C) The two circles A and O have the same radius, and point B lies on the circle, so $AO = AB = BO$. That makes angle BAO equal to $60°$, and by symmetry so does CAO. Then angle BAC is equal to $120°$.

Solution 24. (C) The orthogonal subspace to a 2-dimensional subspace in $\mathbb{R}^4$ is $4 - 2 = 2$-dimensional. Therefore all we need are two vectors orthogonal to the two given that are linearly independent. We can see that (B) is inappropriate but (C) works fine.

Solution 25. (D) Since the partial derivatives of $f(x, y)$ are always nonzero, we achieve the maximum of $f(x, y)$ somewhere on the boundary of the region described. Drawing the region out is fairly helpful. We want to maximise x and minimise y. We actual achieve this at $(2, 0)$, so the maximum is 10.

Solution 26. (B) $f(x)$ is continuous, but it's not differentiable at $x = 1$. We can also see that $f'(0) = f'(2) = 0$. Therefore we can check for the absolute maximum at $x = 0, 1, 2$. We have $f(0) = -2$, $f(1) = 1$, and $f(2) = -2$ as well. That makes $x = 1$ the absolute maximum.

Solution 27. (E) If $f(x) = f(1 - x)$, then $f'(x) = -f'(1 - x)$ by the Chain Rule. Then $f'(0) = -f'(1)$.

Solution 28. (C) We know that $\dim(V_1 + V_2) = \dim V_1 + \dim V_2 - \dim(V_1 \cap V_2)$. The smallest dimension of $V_1 \cap V_2$ is attained when $V_1 + V_2$ is as big as possible, so we must have $\dim(V_1 + V_2) = 10$. That makes $\dim(V_1 \cap V_2) = 2$.

Solution 29. (B) We can use integration by parts. Let $u = x$ and $dv = p''(x)\,dx$. Then

$$\int_0^2 xp''(x)\,dx = xp'(x)\Big|_0^2 - \int_0^2 p'(x)\,dx = 2p'(2) - (p(2) - p(0)) = -2.$$

Solution 30. (D) Everything but (D) is impossible for any basis, regardless of the dimension of V.

Solution 31. (C) We can use the rational root theorem. Any rational root of that polynomial comes in the form of a factor of b over a factor of 9. The only number that we cannot obtain is $1/4$, since 4 and 9 are coprime integers.

Solution 32. (D) First, let's imagine we put Pat dead last. Then we can arrange all the children in front of Pat in any order we want, which gives us 19! combinations. We know that we cannot have 20! combinations, as that's the total number of rearrangements. Thus we want an answer that is strictly less than 20! and strictly more than 19!, making the answer (D). For the actual calculation, we know that in the 20! unrestricted orientations, half of them will have Lynn ahead of Pat and half will have Pat ahead of Lynn, giving us our division by 2.

Solution 33. (A) We know there are $\lfloor 1000/30 \rfloor = 33$ numbers divisible by 30 in that range. The numbers that are divisible by 30 and 16 are exactly the multiples of 240. There are only 4 multiples of 240 in that range, making our answer 29.

Solution 34. (A) Let $\lim_{x\to\infty} f'(x) = L \neq 0$. Then we know that there exists $N \in \mathbb{N}$ such that $|f'(x) - L| < L/2$ for all $x > N$. That is, $3L/2 > f'(x) > L/2$. That would mean that $f(N + M)$ is at least $f(N) + M \cdot L/2$ for all $M > 0$. Since $L \neq 0$, that means that $\lim_{x\to\infty} f(x)$ does not converge. Therefore we must have $\lim_{x\to\infty} f'(x) = 0$.

Solution 35. (B) The tangent plane is those points perpendicular to the gradient of $f(x, y)$ at that point, specifically $\vec{n} = (f_x, f_y, -1)$. We have

$$f_x(x, y) = -e^{-x} \sin y, \quad f_y(x, y) = e^{-x} \cos y \implies f_x(0, \pi/2) = -1, \quad f_y(0, \pi/2) = 0.$$

That makes the equation of the plane $-x - z = c$, or $x + z = c$. There is only one option in this form, so it must be (B). To prove that $c = 1$, just note that $f(0, \pi/2) = 1$, so the point $(0, \pi/2, 1)$ must lie on the plane.

Solution 36. (D) We can calculate these things:

$$\mu(x) = \frac{4 + 9 + 7 + 5 + x}{5} = \frac{25 + x}{5}, \quad \eta(x) = \begin{cases} 5 & x \leq 5 \\ x & x \in (5, 7) \\ 7 & x \geq 7 \end{cases}$$

We also have $\mu(x) = 5+x/5$. We can see that $\mu(x) = \nu(x) = 5$ when $x = 0$, and $\mu(x)\nu(x) = 7$ when $x = 10$. The last potential option is $\mu(x) = \nu(x) = x$, which would mean solving

$$5 + x/5 = x \implies 25 = 4x \implies x = 25/4 = 6.25.$$

Indeed, this x falls in $(5, 7)$, giving us three solutions.

Solution 37. (B) First, we can simplify this series to

$$\sum_{k=1}^{\infty} \frac{k^2}{k!} = \sum_{k=1}^{\infty} \frac{k}{(k-1)!} = \sum_{k=0}^{\infty} \frac{k+1}{k!} = \sum_{k=0}^{\infty} \frac{k}{k!} + \sum_{k=0}^{\infty} \frac{1}{k!}.$$

The final term in the above line is equal to e. The first term of the penultimate sum is 0, so we obtain

$$\sum_{k=0}^{\infty} \frac{k}{k!} = \sum_{k=1}^{\infty} \frac{k}{k!} = \sum_{k=1}^{\infty} \frac{1}{(k-1)!} = \sum_{k=0}^{\infty} \frac{1}{k!} = e.$$

Therefore the total sum is $2e$.

Solution 38. (B) We can just do some calculations. (A) computes to $1 - \sqrt{2}/2$ and (B) computes to $\sqrt{2}/2$. (D) computes to $1/2$. (C) we can calculate using

$$2 \cos^2 x - 1 = \cos 2x \implies \cos^2 x = \frac{\cos 2x + 1}{2}.$$

That makes the integral $1/4 + \pi/8$ (working it all out). Then also using that $\sin x \cos x = \frac{\sin 2x}{2}$, we get that (E) is $1/4$. Since $\sqrt{2}/2 \approx 0.7$ and $1/4 + \pi/8 \approx 0.6$, we see that (B) is the biggest.

Solution 39. (B) Let us examine each option. (A) is the right Riemann sum, (B) is the left Riemann sum, and (C) is the midpoint sum. (D) is the actual integral and (E) is 0. We know that the integral will be bigger than zero, so we can rule that out. Moreover, we know that e^{-x} on $[0, 10]$ is strictly decreasing and concave up. That means that the left Riemann sum is the biggest overestimate of the integral.

Solution 40. (E) I believe this is the easier way to compute this. We know that $P(H > 4) = P(H < 4)$ by symmetry. Therefore our answer should be half of $1 - P(H = 4)$. This is one computation:

$$P(H = 4) = \binom{8}{4}\left(\frac{1}{2}\right)^8 = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 2^8} = \frac{35}{128}.$$

Therefore our answer should be $93/256$.

Solution 41. (E) Let's find the gradient.

$$f_x = y - 3x^2, \quad f_y = x - 3y^2.$$

Trying to solve for $f_x = 0$, this gives us $y = 3x^2$. Putting this into $f_y = 0$ gives $x - 27x^4 = 0$ so $x(1 - 27x^3) = 0$, so $x = 0$ or $x = 1/3$. That makes $y = 0$ or $y = 1/3$ as well, so we must test $(0, 0)$ and $(1/3, 1/3)$. We also have

$$f_{xx} = -6x, \quad f_{xy} = f_{yx} = 1, \quad f_{yy} = -6y.$$

That makes the determinant of the Hessian $D(x, y) = 36xy - 1$. Then $D(0, 0) = -1$, so there we have a saddle point. But $D(1/3, 1/3) = 3 > 0$,, and $f_{xx}(1/3, 1/3) = -2 < 0$, which gives us a local maximum.

Solution 42. (D) We know that AC or BC will be one side of the parallelogram. The last point of the parallelogram is given by the choice of AC, BC, or both AC and BC as sides. The first two options make BC or AC (respectively) the diagonal of the parallelogram, while the third implies that AB is a diagonal. There are three total options.

Solution 43. (B) The vector $(6, 7, 8)$ is represented by $2 \cdot (3, 4, 5) - 1 \cdot (0, 1, 2)$. Therefore $A(6, 7, 8) = 2(0, 1, 0) - (1, 0, 0) = (-1, 2, 0)$.

Solution 44. (D) We could also write this $f(x) = x^{x/2}$, so that makes (C) true. (B) is also clearly true. It is also easy to convince ourselves that (A) is probably true, so we should take a derivative. Using logarithmic derivation, we have

$$\log f(x) = \frac{x}{2}\log x \implies (\log f(x))' = \frac{x}{2} \cdot \frac{1}{x} + \log x \cdot \frac{1}{2} = \frac{1 + \log x}{2}.$$

That makes

$$f'(x) = f(x) \cdot (\log f(x))' = \frac{x^{x/2}(1 + \log x)}{2}.$$

This is strictly increasing (as it is a product of strictly increasing functions), but is definitely not always positive. As $x \to 0$, we have $\log x \to -\infty$, making $f'(x) < 0$ in particular.

Solution 45. (A) We need to integrate gallons per hour over that four hour range. If $E(v)$ is in miles per gallon and $v(t)$ is in miles per hour, then to get the right units we need to take $v(t)/E(v(t))$. None of the other units make sense, so (A) must be right.

Solution 46. (C) We can start calculating these eigenvalues.

$$\det \begin{bmatrix} \cos t - \lambda & -\sin t \\ \sin t & \cos t - \lambda \end{bmatrix} = (\lambda - \cos t)^2 + \sin^2 t = \lambda^2 - 2\cos t\lambda + 1.$$

We don't need to solve any further, but we know that $\lambda_1 + \lambda_2 = 2\cos t$. Our desired value gives us $\cos t = 1/2$, so $t = \pi/3$.

Solution 47. (E) We can visualise this as the square $[0,1]^2$. We need to satisfy $|x-y| < 1/2$. If $x > y$, this means that $x - y < 1/2$, so $y > x - 1/2$. If $x < y$, then we have $y - x < 1/2$, so $y < x + 1/2$. This gives us the entire square but for two triangles of base and height $1/2$. The total area not satisfying $|x - y| < 1/2$ is $1/4$, so the probability is $3/4$.

Solution 48. (A) For the variable v, we should expect values only in $[1, 2]$. That narrows our options down to (A) or (D). These only differ on their sides, not their corners. We know that the parametric equation $x = 0, y = t$ for $t \in [0, 1]$ is the left boundary of the square. If we apply the transformation into u, v variables, we obtain $u = 0 + t$ and $v = 1 + t$. That gives the total equation $v = u+1$, which is certainly not the same as $v = 1+u^{1/3}$. Therefore (D) cannot be the answer.

Solution 49. (E) I is true because it is just a u-substitution. II is true, more clear once we write $-\int_b^3 f(x)\,dx = \int_3^b f(x)\,dx$. III is also a u-substitution, and is also true.

Solution 50. (D) We certainly know that $f(x) = x$ and $f(x) = -x$ are valid options. However, we could also have $f(x) = -|x|$ and $f(x) = |x|$, bringing the total options up to 4.

Solution 51. (D) Using the ratio test, we need

$$\frac{(x+2y)^{k+1}}{k+1} \cdot \frac{k}{(x+2y)^k} = (x+2y) \cdot \frac{k}{k+1}.$$

Taking the limit as $k \to \infty$, this gives us $|x + 2y| < 1$ as our radius of convergence. That makes the two lines $x + 2y = 1$ and $-x - 2y = 1$ the boundaries, which are parallel.

Solution 52. (D) We can phrase this better in terms of the matrix implied by the coefficients of the equations. I is not true because the solution $(0, 0, 0)$ is always possible. For II to be true, the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & b \end{pmatrix}$$

would need nonzero determinant. There is certainly a value of b for which this is true – it will be true for most values of b, since the first two rows are linearly independent. Finally, III is true if the matrix has determinant zero, which happens if $b = 5$.

Solution 53. (D) This integral is calculable by using the residue theorem. The integral of $f(z)$ around C is given by $2\pi i$ times the residues of any poles inside of C. This is only the point $z = 1$, as $z = -3$ does not lie inside C. Since this is a simple pole, we can calculate the residue by

$$\lim_{z\to 1}(z-1) \cdot \frac{1}{(z-1)(z+3)^2} = \lim_{z\to 1} \frac{1}{(z+3)^2} = \frac{1}{16}.$$

That makes the value of the integral $2\pi i/16 = \pi i/8$.

Solution 54. (A) We need to solve the differential equation

$$h'(t) = 1 - 0.25h(t).$$

The limit value would be when $h'(t) = 0$, which we can see occurs at $h(t) = 4$. Our tank is a cube with 10 feet on each side, so the volume in the rank when the height of the water is 4 gives us 400 cubic feet.

Solution 55. (D) We are given in the problem that $f'(0) < 0$, so f needn't be increasing on the interval $[0, \infty)$. If $f''(x)$ is increasing, then at some point $f''(0)$ will become positive, which will make $f'(x)$ go from negative to positive. At that point $f(x)$ will be strictly increasing, giving us a unique zero and making $f(x)$ unbounded.

Solution 56. (E) Our best options look like (D) or (E). One of these will satisfy the triangle inequality, and the other will not necessarily. If we know that $d(x, y) + d(y, z) \geq d(x, z)$, which equality for some choices of (x, y, z). If we square both sides, we get

$$d(x, y)^2 + 2d(x, y)d(y, z) + d(y, z)^2 = d(x, z)^2.$$

Since this middle term is not trivial, we do not know that $d^2(x, y) + d^2(y, z) \geq d^2(x, z)$ for choices when we have equality above. However, if we take the square root,

$$\sqrt{d(x, y) + d(y, z)} \geq \sqrt{d(x, z)}.$$

We further know that $\sqrt{d(x, y)} + \sqrt{d(y, z)} \geq \sqrt{d(x, y) + d(y, z)}$, so indeed $\sqrt{d}$ still satisfies the triangle inequality.

Solution 57. (C) I is a subring of $\mathbb{R}[x]$. It is an abelian group under addition, and it is still closed under multiplication. II, however, is not. It is not closed under addition: $(x^2 + x + 1) - (x^2 + 1) = x$, which has odd degree now. $\mathbb{Q}[x] \subset \mathbb{R}[x]$ is also a subring, since $\mathbb{Q} \subset \mathbb{R}$ is a subring.

Solution 58. (C) The continuous image of a connected subset is connected. However, we needn't get an open subset. If $f(x)$ is a constant function, then S is a singleton set. Since we know that $f(x)$ is defined not just on $(0, 1)$ but on all of $\mathbb{R}$. That means that $f([0, 1])$ is a bounded set, as it is the continuous image of a compact set. This makes S bounded as well.

Solution 59. (A) If this set has only two elements, then $x^3 = x^5$, $x^5 = x^9$, or $x^3 = x^9$. If the first holds, then $x^2 = 1$, so this set has only the element x in it. If the second holds, then $x^4 = 1$. If the third holds, then $x^6 = 1$. However, we know that x is an element of the cyclic group of order 15, so $|x|$ must divide 15. Therefore we must actually have $x^3 = 1$. That makes $x^{13} = x$, so the number of elements of the set is equal to the order of x.

Solution 60. (E)[1] If $s = s^2$, then we know that

$$2s = s + s = (s + s)^2 = 4s^2 = 4s \implies 2s = 0$$

so that $s + s = 0$ for all $s \in S$. Second,

$$s + t = (s + t)^2 = s^2 + st + ts + t^2 = s + st + ts + t$$

[1]Thank you to Michael Tong for emailing me a correction.

by using $t = t^2$ and $s = s^2$. Rearranging, this gives us $st + ts = 0$, hence $st = -ts$. It would appear that S is now anticommutative, but by the above we know that $-ts = ts$ hence S is commutative, proving III. But now because S is commutative, $(s+t)^2 = s^2 + 2st + t^2$, but this middle term is zero by I so $(s+t)^2 = s^2 + t^2$, proving II.

Solution 61. (E)[2] We can factor this as $(p^2+1)(p+1)(p-1)$. We can start counting up the factors that this necessarily has. We know that any $p > 5$ is odd, so p^2+1, $p+1$, and $p-1$ are all divisible by 2. Further, one of $p+1$ and $p-1$ is divisible by 4, so this gives a factor of 16. We know also that $p \equiv 1, 2 \mod 3$, so $p^4 - 1 \equiv 0 \mod 3$. Finally, we know by Fermat's little theorem that $p^4 \equiv 1 \mod 5$, so 5 is another factor. That gives a total of $16 \cdot 3 \cdot 5 = 240$.

Solution 62. (A) We can work out by hand that $(1+x)^3 = 1 + 3x + 3x^2 + x^3$. We also know that

$$(2+x^2)^{10} = 2^{10} + 2^9 \cdot 10 \cdot x^2 + Ax^4$$

for some large polynomial A. Then the x^3 term of the final product is given by the product of the x^3 term of the first with the constant term of the second, and the x term of the first with the x^2 term of the second. That gives us $2^{10} + 2^9 \cdot 30 = 2^9(32) = 2^{14}$.

Solution 63. (D) Judging by the basic shape of the graphs, we should expect two solutions, one negative and one positive around $x = 0$. But then we know that 2^x grows faster than x^{12}, so there should be some large $x > 0$ for which the graphs intersect, giving three total.

Solution 64. (C) I is the claim that the image of $[0,1]$ is bounded under a continuous function. This is true: it is compact, so must be bounded. II is the statement of continuity for the choice $\varepsilon = 1$, $D = \delta$. But III is the claim that f is Lipschitz continuous, which may not be the case. On a compact set, this would imply that f is differentiable almost everywhere. The Weierstrass function, to use a powerful example, would not satisfy this.

Solution 65. (A) We see that $x = -3$ is a simple root of $p(x)$. We know the constant term of $p(x)$ is equal to the negation of the product of all the roots. If we had a sense of where the last root was, we could find c. We know that the graph turns twice, being a cubic function with 3 real roots, and furthermore since $p'(-3) < 0$ the graph turns for the first time on $(-\infty, -3)$. Therefore there is a root somewhere in that range, so $c = 6 \cdot \alpha$ for some $\alpha < -3$. That means $c < -18$, so that makes $c = 2 - 27$ the only option.

Solution 66. (E) Once again, a classic Green's theorem problem. We know that

$$\oint_C -2y\,dx + x^2\,dy = \iint_D 2x + 2\,dxdy.$$

We know that the 2 part of this integral gives us twice the area of the disc, which is 18π. The $2x$ part will integrate to zero, since it is an odd function on a symmetric domain.

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[2]See here